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The Error Modelling for Rolling Surfaces Generation I. Algorithms

Ş. L. ing. Virgil TEODOR, prof. dr. ing. Nicolae OANCEA
Universitatea „Dunărea de Jos" din Galaţi

SUMMARY

The generation of profiles by enwrapping with tools on type rack-gear or gear-cutter is obviously affected by the errors of execution of these tool's profiles, which can not be eliminated, these representing a theoretical errors of engendering. In this paper, is proposed an algorithm for the determination of the effective profile shape of the piece, regarding the "measured" errors of the tool's profile.

Keywords: *errors, modelling, rolling centroids, the in-plane trajectory method*

1. Introduction

The realization of the cutting tools, in most cases is affected by errors which distort the shape and dimensions of the tool's profile [1], [5], [7], [8], [9].

As result, first possibility to erroneous generate the surfaces obtained by enwrapping is due to the geometrical error of the real profile of the tool.

As a fundamental error, the geometrical error may lead, in some cases at shape and dimension errors big enough so the generating surface to be beside of the tolerances field.

So, the numerical modelling of the geometrical error of the cutting edge of the tool may be a way to estimate the theoretical level of the error of the surface to be generated [5], [6].

2. The surface error modelling

We propose, in the following, an algorithm for generation with tools associated with rolling centroids modelling based on the "in-plane trajectory method" [2], [4], as method for study the reciprocally enwrapping profiles, equivalent with the Gohman theorem, the fundamental theorem of the reciprocally enwrapping surfaces [1].

2.1. Generation with rack-gear tool

In figure 1, are showed the actual profile of the rack-gear tool and the references systems associated with the two rolling centroids.

xy is the fixed reference system;

XY –reference system associated with C1 centroid;

ξη –reference system associated with C2 centroid.

The, real profile -C_{SC}, of the rack-gear tool, obtained by the measuring of the tool, may be expressed by a matrix on form

$$C_{SC} = \left\| \begin{matrix} \xi_1\eta_1 \\ \xi_2\eta_2 \\ \vdots \\ \xi_i\eta_i \\ \xi_{i+1}\eta_{i+1} \\ \vdots \\ \xi_n\eta_n \end{matrix} \right\| \qquad (1)$$

where the "n" number of point depends to the precision with whence the theoretical profile of tool is known.

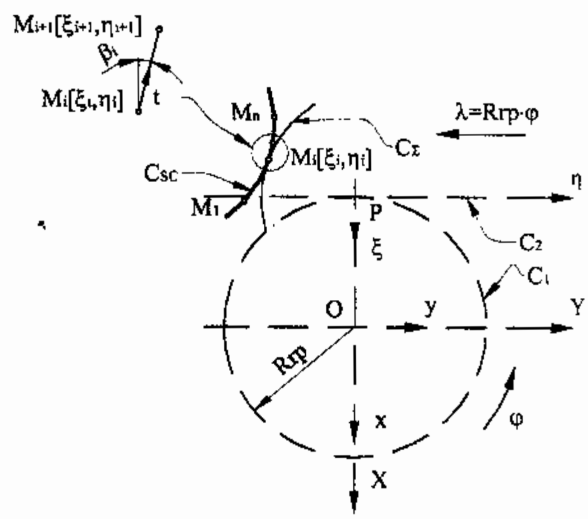

Fig. 1. Generation with rack-gear tool

The appliance of the in-plane trajectory method for the numerical modelling of the theoretical profile presumes the analytical know of the profile that enveloping is searching, which, in this case is not possible.

On this line, we propose the linearization [2], [5] of the actually profile between two successive points, $M_i[\xi_i,\eta_i]$, $M_i[\xi_{i+1},\eta_{i+1}]$, so, on segments, the actually tool's profile will be regarded as an analytical profile with equations:

$$M_{i,i+1}\left|\begin{aligned}&\xi=\xi_i-t\cdot\cos\beta_i;\\&\eta=\eta_i+t\cdot\sin\beta_i;\end{aligned}\right. \qquad tg\beta_i=\frac{|\eta_{i+1}-\eta_i|}{|\xi_{i+1}-\xi_i|}, \tag{2}$$

with t continuous variable.

In this way, on parts, the actually profile of the tool is expresed in analytical form, making easy the appliance of the in-plane trajectory method for the enveloping study.

Knowing the relative motion

$$X=\omega_3(\varphi)[\xi+a] \tag{3}$$

of the C_1 and C_2 centroids, as so as the relative motion of the mobile reference systems associated with these, is determined the trajectories family $(T)_\varphi$ points belongs to the elementary profile M_iM_{i+1},

$$\begin{Vmatrix}X\\Y\end{Vmatrix}=\begin{Vmatrix}\cos\varphi & \sin\varphi\\-\sin\varphi & \cos\varphi\end{Vmatrix}\cdot\begin{Vmatrix}\xi_i-t\cos\beta_i-R_{rp}\\\eta_i+t\sin\beta_i-R_{rp}\cdot\varphi\end{Vmatrix} \tag{4}$$

or, in developed form:

$$(T)_\varphi\left|\begin{aligned}&X=\left[\xi_i-t\cos\beta_i-R_{rp}\right]\cos\varphi+\\&\quad+\left[\eta_i+t\sin\beta_i-R_{rp}\varphi\right]\sin\varphi;\\&Y=-\left[\xi_i-t\cos\beta_i-R_{rp}\right]\sin\varphi+\\&\quad+\left[\eta_i+t\sin\beta_i-R_{rp}\varphi\right]\cos\varphi.\end{aligned}\right. \tag{5}$$

The trajectories family's is associated the enwrapping condition, in specifically form:

$$\frac{X'_\varphi}{X'_t}=\frac{Y'_\varphi}{Y'_t} \tag{6}$$

where

$$\begin{aligned}&X'_t=-\cos(\varphi+\beta_i);\\&Y'_t=\sin(\varphi+\beta_i);\\&X'_\varphi=-\xi_i\sin\varphi+\eta_i\cos\varphi+\\&+t\sin(\varphi+\beta_i)-R_{rp}\varphi\cos\varphi;\\&Y'_\varphi=-\xi_i\cos\varphi-\eta_i\sin\varphi+\\&+t\cos(\varphi+\beta_i)+R_{rp}\varphi\sin\varphi.\end{aligned} \tag{7}$$

The (5), (6) and (7) equations assembly determine the enveloping of the M_iM_{i+1} part of the actual tool's profile, idle the actual profile generated corresponding to this segment.

The assembly of all these segments is the profile generated by the tool's profile.

The limits for "t" parameter on the straight elementary profile of the tool are:

$$\begin{aligned}&t_{min}=0;\\&t_{max}=\sqrt{(\xi_i-\xi_{i+1})^2+(\eta_i-\eta_{i+1})^2}.\end{aligned} \tag{8}$$

2.2. Generation with gear-cutter tool

Similarlly, see figure 2, is approach the problem of the geneation with gear-cutter tool.

The references systems are:

xy and x_0y_0 fixed references systems;

ξη and XY mobile reference systems associated with C_1 and C_2 centroids.

The elementary profile, the substitute for the tool's actual profile has the equations (2), with t variable parameter.

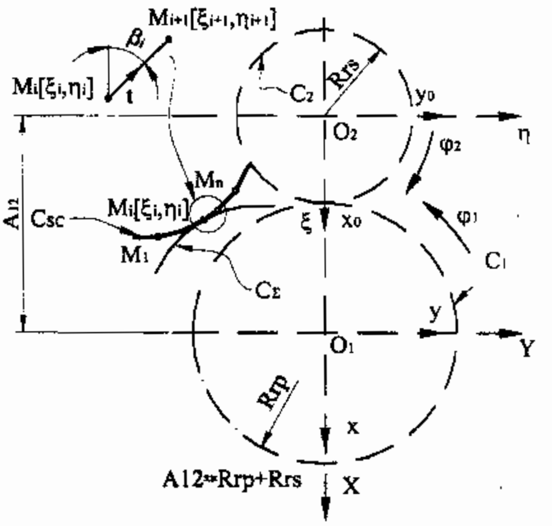

Fig. 2. Generation with gear-cutter tool

In the relative motion of the two rolling centroids, with gear ratio

$$\varphi_2=\varphi_1\frac{R_{rp}}{R_{rs}}, \tag{9}$$

defined by transformation

$$X=\omega_3(\varphi_1)\left[\omega_3^T(-\varphi_2)\xi+a\right] \tag{10}$$

they are defined the point's trajectories $(T)_{\varphi_1}$ which belongs of the $M_i[\xi_i,\eta_i]\,M_{i+1}[\xi_{i+1},\eta_{i+1}]$

$$(T)_{\varphi_1}:\begin{Vmatrix}X\\Y\end{Vmatrix}=\begin{Vmatrix}\cos\varphi_1 & \sin\varphi_1\\-\sin\varphi_1 & \cos\varphi_1\end{Vmatrix}\cdot\left[\begin{Vmatrix}\cos\varphi_2 & \sin\varphi_2\\-\sin\varphi_2 & \cos\varphi_2\end{Vmatrix}\cdot\begin{Vmatrix}\xi_i+t\cos\beta_i\\\eta_i-t\sin\beta_i\end{Vmatrix}+\begin{Vmatrix}-A_{12}\\0\end{Vmatrix}\right]. \tag{11}$$

The enwrapping condition of the (11) trajectories is

$$\frac{X'_{\varphi_1}}{X'_t}=\frac{Y'_{\varphi_1}}{Y'_t} \tag{12}$$

where:

$$\begin{aligned}
&X_t' = \cos\left[(1+i)\varphi_1 + \beta_i\right];\\
&Y_t' = -\sin\left[(1+i)\varphi_1 + \beta_i\right];\\
&X_{\varphi_1}' = -(1+i)(\xi_i + t\cos\beta_i)\sin(1+i)\varphi_1 +\\
&\quad + (1+i)(\eta_i - t\sin\beta_i)\cos(1+i)\varphi_1 +\\
&\quad + A_{12}\sin\varphi_1;\\
&Y_{\varphi_1}' = -(1+i)(\xi_i + t\cos\beta_i)\cos(1+i)\varphi_1 -\\
&\quad - (1+i)(\eta_i - t\sin\beta_i)\sin(1+i)\varphi_1 +\\
&\quad + A_{12}\cos\varphi_1.
\end{aligned} \tag{13}$$

2.3. Generation with rotary cutter tool

In figure 3, are showed the two centroids associated with the blank' axial section and with the rotary cutter tool, as so as the reference systems associated with these:

xy is the fixed reference system;

XY and ξη are the mobiles references systems, associated with C_1 and C_2 centroids.

The tool's elementary profile is described by equations (2).

In the relative motion

$$X = \omega_3^T(\varphi)\xi - a;\qquad a = \begin{Vmatrix} -R_{rs} \\ -R_{rs}\varphi \end{Vmatrix}, \tag{14}$$

is determinated the trajectorie's family

$$(T)_\varphi : \begin{Vmatrix} X \\ Y \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{Vmatrix} \begin{Vmatrix} \xi_i - t\cos\beta_i \\ \eta_i + t\sin\beta_i \end{Vmatrix} - \begin{Vmatrix} -R_{rs} \\ -R_{rs}\varphi \end{Vmatrix}. \tag{15}$$

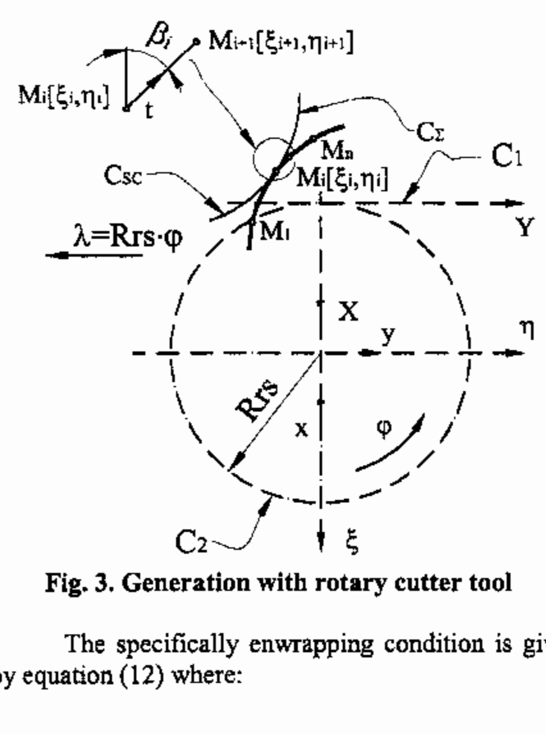

Fig. 3. Generation with rotary cutter tool

The specifically enwrapping condition is give by equation (12) where:

$$\begin{aligned}
&X_t' = -\cos(\varphi - \beta_i);\\
&Y_t' = -\sin(\varphi - \beta_i);\\
&X_\varphi' = -\xi_i\sin\varphi - \eta_i\cos\varphi -\\
&\quad - t\sin(\varphi - \beta_i);\\
&Y_\varphi' = \xi_i\cos\varphi - \eta_i\sin\varphi +\\
&\quad + t\cos(\varphi - \beta_i) + R_{rs}.
\end{aligned} \tag{16}$$

The real profile generated by tool is determined associating with (15) equations the (2) enwrapping condition.

The limits of „t" parameters are give by (8) equations.

If the number of the measured points along the tool's profile (rack-gear, gear-cutter or rottary cutter tool) is big enough, so the segment length $\delta = \overline{M_i M_{i+1}}$ is small enough,

$$\delta = \sqrt{(\xi_i - \xi_{i+1})^2 + (\eta_i - \eta_{i+1})^2}, \tag{17}$$

(as example $\delta \le 1\cdot 10^{-1}$ mm), then, the specifically enveloping condition (see (6), (12)) they can be particularised, for t=0, having the form (see (12))

$$\frac{-\xi_i\sin\varphi + \eta_i\cos\varphi - R_{rp}\varphi\cos\varphi}{-\cos(\varphi + \beta_i)} = \frac{-\xi_i\cos\varphi - \eta_i\sin\varphi + R_{rp}\varphi\sin\varphi}{\sin(\varphi + \beta_i)} \tag{18}$$

or, similarlly (see (13))

$$\frac{\cos\left[(1+i)\varphi_1 + \beta_i\right]}{-(1+i)\left[\xi_i\sin(1+i)\varphi_1 - \eta\cos(1+i)\varphi_1\right] + A_{12}\sin\varphi_1} = \frac{-\sin\left[(1+i)\varphi_1 + \beta_i\right]}{-(1+i)\left[\xi_i\cos(1+i)\varphi_1 + \eta\sin(1+i)\varphi_1\right] + A_{12}\cos\varphi_1}. \tag{19}$$

4. Conclusions

With this algorithm is possible to calculate the errors due of the geometrical errors of the tool's cutting edge.

This error modelling is a way to predict the theoretical error of the generated profile so this errors to not drive to dimensions which exceed the tolerance limit of the profile.

Based of this algorithm it was make special software which may calculate the errors obtained with a tool of which profile is known.

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Rezumat

Generarea profilurilor prin înfăşurare cu scule de tip cremalieră sau cuţit-roată este influenţată, evident, de erorile de execuţie ale profilurilor acestor scule, erori geometrice care nu pot fi eliminate, acestea reprezentând o eroare teoretică de generare.

În acest sens, în prima parte a acestei lucrări (I. Algorithms), se propune un algoritm pentru stabilirea formei profilului efectiv generat al semifabricatului, ţinând seama de eroarea „măsurată" a profilului sculei-cremalieră sau a cuţitului-roată.

În partea a doua a lucrării (II. Applications) sunt prezentate exemple numerice, în baza unor produse soft originale, pornind de la modelarea unor limite de eroare ale sculelor, pentru un tip anume de profil de generat, în baza metodei „traiectoriilor plane" [2], [3], [4].

Résumé

La realization des profiles d'outils est dans le majorité des cases affecté par erreurs qui est influente par les erreurs d'executions du cette outils, erreurs géométriques, qui non peut etre elimine, cette représente une erreur théoretique de génération.

Dans ce sens, est propose, un algorithm pour déterminé la forme du profile génére (I. Algorithms).

Dans le second part du cette ouvrage (II. Applications), ils sont presentes examples numeriques, réalisé avec un logiciel écrit pour un certain profile de génére, dans la base du method des trajectories planes [2], [3]. [4].